Problems

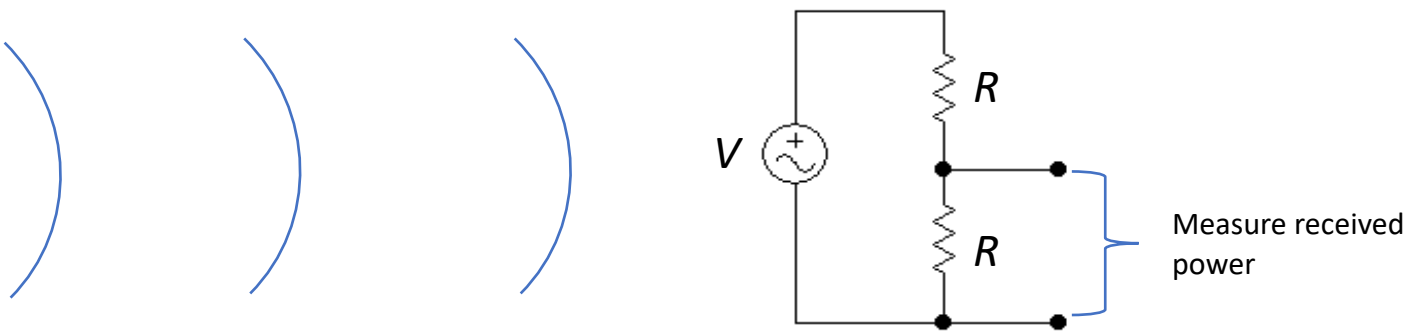
1. (20 pts) Show that the radiation resistance of the Hertzian dipole radiator is given by,

$$R_{Hertz} = \frac{2\pi}{3}\,\mu_0 c\,\left(\frac{d}{\lambda}\right)^2 = \frac{2\pi}{3}\,(120\,\pi)\,\left(\frac{d}{\lambda}\right)^2 = 80\,\pi^2\,\left(\frac{d}{\lambda}\right)^2$$

2. (20 pts) Show that a Hertzian dipole of length *d* has an effective area of $A_{eff} = 3\,\lambda^2/8\pi$. Assume:

a. The antenna receiving power acts like a sinusoidal voltage generator whose voltage amplitude is *V* = *Ed* where *E* is the amplitude of the electromagnetic wave arriving at the antenna.

b. The effective circuit is shown below. *V* is the voltage generator corresponding to the radiation received by the antenna. *R* is the radiation resistance calculated in problem 1. To obtain maximum power the antenna must be impedance-matched to a real resistor of the same value *R* across which we measure the signal.

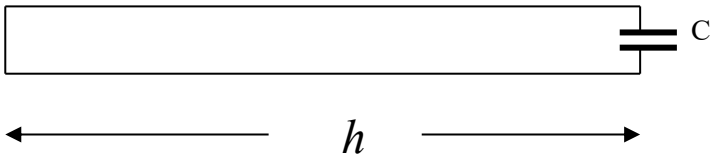


3. (15 pts) Show that the maximum gain of the Hertzian dipole antenna is 3/2. This, together with the result from problem 2 shows that,

$$\frac{A_{eff}}{Gain(maximum)} = \frac{\lambda^2}{4\pi}$$

This result is true for antennas in general, not just the Hertzian dipole.

4. (20 pts) A transmission line has capacitance per unit length C_0, inductance per unit length L_0 and length h. The line is shorted-circuit at one end. The other end is terminated with a capacitor of value C. Assume there is no resistance in the circuit and denote the characteristic impedance by Z_0.

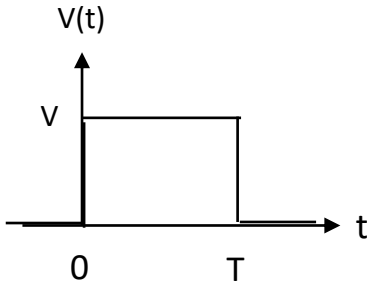


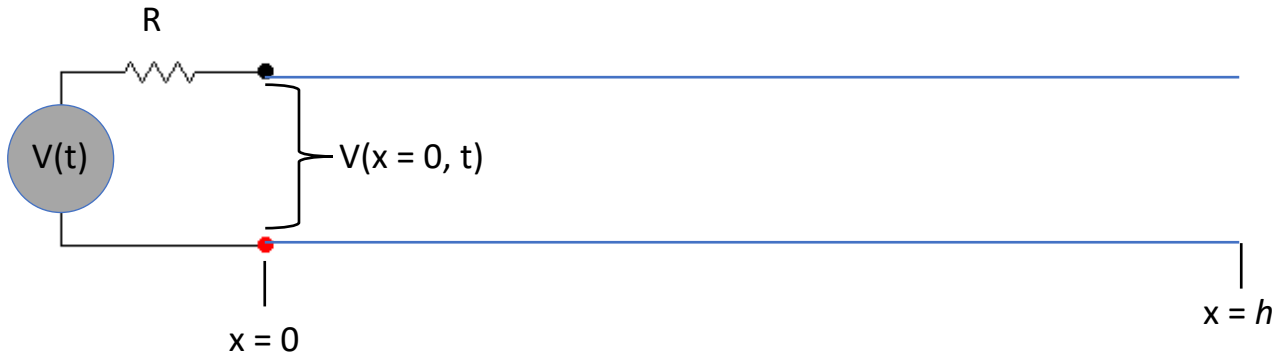
Show that this system has standing wave oscillations at frequencies ω that obey,

$$\frac{1}{\omega} = Z_0 C \tan\left(\omega h \sqrt{L_0 C_0}\right)$$

Use a graphical solution of this equation to indicate the oscillation frequencies.

5. (20 pts) A generator is connected through a resistor *R* to a transmission line of length *h* = 3 m. The transmission line has no dielectric, characteristic impedance *R* and is open circuited at x = *h*. The generator creates a step of voltage V for a duration T = 10^{-3} sec. . Draw the voltage
V(x = 0, t) versus time. Assume that everything to the left of x = 0 has negligible size compared to *h*. Explain, using left and right going waves, how the voltage takes the form that it does. Hint: treat the square pulse as the sum of a positive step function and a time-delayed negative step function and use superposition.





6. (5 pts) An electron is travelling through turpentine, whose index of refraction *n* = 1.47. Find the minimum electron kinetic energy, in keV, to generate Cerenkov radiation. You'll need special relatively to do this.